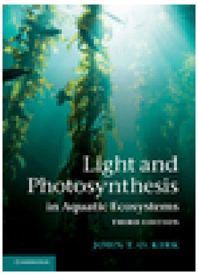

Light and Photosynthesis in Aquatic Ecosystems

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Chapter

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1
Concepts of hydrologic optics

1.1 Introduction

The purpose of the first part of this book is to describe and explain the behaviour of light in natural waters. The word 'light' in common parlance refers to radiation in that segment of the electromagnetic spectrum – about 400 to 700 μm to which the human eye is sensitive. Our prime concern is not with vision but with photosynthesis. Nevertheless, by a convenient coincidence, the waveband within which plants can photosynthesize corresponds approximately to that of human vision and so we may legitimately refer to the particular kind of solar radiation with which we are concerned simply as 'light'.

Optics is that part of physics which deals with light. Since the behaviour of light is greatly affected by the nature of the medium through which it is passing, there are different branches of optics dealing with different kinds of physical systems. The relations between the different branches of the subject and of optics to fundamental physical theory are outlined diagrammatically in Fig. 1.1. Hydrologic optics is concerned with the behaviour of light in aquatic media. It can be subdivided into limnological and oceanographic optics according to whether fresh, inland or salty, marine waters are under consideration. Hydrologic optics has, however, up to now been mainly oceanographic in its orientation.

1.2 The nature of light

Electromagnetic energy occurs in indivisible units referred to as *quanta* or *photons*. Thus a beam of sunlight in air consists of a continual stream of photons travelling at $3 \times 10^8\,m\,s^{-1}$. The actual numbers of quanta

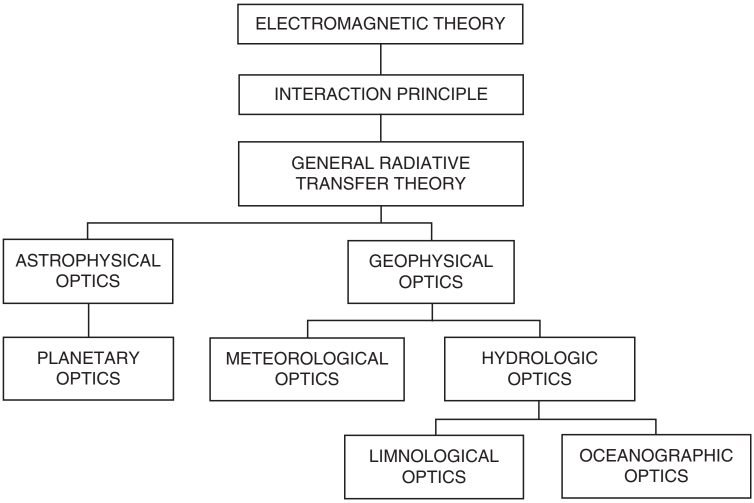

Fig. 1.1 The relationship between hydrologic optics and other branches of optics (after Preisendorfer, 1976).

involved are very large. In full summer sunlight. for example, 1 m^2 of horizontal surface receives about 10^{21} quanta of visible light per second. Despite its particulate nature, electromagnetic radiation behaves in some circumstances as though it has a wave nature. Every photon has a wavelength, λ, and a frequency, v. These are related in accordance with

$$\lambda = c/v \tag{1.1}$$

where c is the speed of light. Since c is constant in a given medium, the greater the wavelength the lower the frequency. If c is expressed in m s^{-1} and v in cycles s^{-1}, then the wavelength, λ, is expressed in metres. For convenience, however, wavelength is more commonly expressed in nanometres, a nanometre (nm) being equal to 10^{-9} m. The energy, ε, in a photon varies with the frequency, and therefore inversely with the wavelength, the relation being

$$\varepsilon = hv = hc/\lambda \tag{1.2}$$

where h is Planck's constant and has the value of 6.63×10^{-34} J s. Thus, a photon of wavelength 700 nm from the red end of the photosynthetic spectrum contains only 57% as much energy as a photon of wavelength 400 nm from the blue end of the spectrum. The actual energy in a photon of wavelength λ nm is given by the relation

$$\varepsilon = (1988/\lambda) \times 10^{-19}\ \text{J} \tag{1.3}$$

A monochromatic radiation flux expressed in quanta s^{-1} can thus readily be converted to $J\,s^{-1}$, i.e. to watts (W). Conversely, a radiation flux, Φ, expressed in W, can be converted to quanta s^{-1} using the relation

$$\text{quanta s}^{-1} = 5.03\ \Phi\lambda \times 10^{15} \tag{1.4}$$

In the case of radiation covering a broad spectral band, such as for example the photosynthetic waveband, a simple conversion from quanta s^{-1} to W, or *vice versa*, cannot be carried out accurately since the value of λ varies across the spectral band. If the distribution of quanta or energy across the spectrum is known, then conversion can be carried out for a series of relatively narrow wavebands covering the spectral region of interest and the results summed for the whole waveband. Alternatively, an approximate conversion factor, which takes into account the spectral distribution of energy that is likely to occur, may be used. For solar radiation in the 400 to 700 nm band above the water surface, Morel and Smith (1974) found that the factor (*Q*/*W*) required to convert W to quanta s^{-1} was 2.77×10^{18} quanta $s^{-1}\,W^{-1}$ to an accuracy of plus or minus a few per cent, regardless of the meteorological conditions.

As we shall discuss at length in a later section (§6.2) the spectral distribution of solar radiation under water changes markedly with depth. Nevertheless, Morel and Smith found that for a wide range of marine waters the value of *Q*:*W* varied by no more than ±10% from a mean of 2.5×10^{18} quanta $s^{-1}\,W^{-1}$. As expected from eqn 1.4, the greater the proportion of long-wavelength (red) light present, the greater the value of *Q*:*W*. For yellow inland waters with more of the underwater light in the 550 to 700 nm region (see §6.2), by extrapolating the data of Morel and Smith we arrive at a value of approximately 2.9×10^{18} quanta $s^{-1}\,W^{-1}$ for the value of *Q*:*W*.

In any medium, light travels more slowly than it does in a vacuum. The velocity of light in a medium is equal to the velocity of light in a vacuum, divided by the refractive index of the medium. The refractive index of air is 1.00028, which for our purposes is not significantly different from that of a vacuum (exactly 1.0, by definition), and so we may take the velocity of light in air to be equal to that in a vacuum. The refractive index of water, although it varies somewhat with temperature, salt concentration and wavelength of light, may with sufficient accuracy he regarded as equal to 1.33 for all natural waters. Assuming that the velocity of light in a vacuum is $3 \times 10^8\,\mathrm{m\,s^{-1}}$, the velocity in water is therefore about $2.25 \times 10^8\,\mathrm{m\,s^{-1}}$ The frequency of the radiation remains the same in water but the wavelength diminishes in proportion to the decrease in velocity. When referring to monochromatic radiation, the wavelength we shall attribute to it is that which it has in a vacuum. Because c and λ change in parallel, eqns 1.2, 1.3 and 1.4 are as true in water as they are in a vacuum: furthermore, when using eqns 1.3 and 1.4. it is the value of the wavelength in a vacuum which is applicable, even when the calculation is carried out for underwater light.

1.3 The properties defining the radiation field

If we are to understand the ways in which the prevailing light field changes with depth in a water body, then we must first consider what are the essential attributes of a light field in which changes might be anticipated. The definitions of these attributes, in part, follow the report of the Working Groups set up by the International Association for the Physical Sciences of the Ocean (1979), but are also influenced by the more fundamental analyses given by Preisendorfer (1976). A more recent account of the definitions and concepts used in hydrologic optics is that by Mobley (1994).

We shall generally express direction within the light field in terms of the *zenith angle,* θ (the angle between a given light pencil, i.e. a thin parallel beam, and the upward vertical), and the *azimuth angle,* ϕ (the angle between the vertical plane incorporating the light pencil and some other specified vertical plane such as the vertical plane of the Sun). In the case of the upwelling light stream it will sometimes be convenient to express a direction in terms of the *nadir angle*, θ_n (the angle between a given light pencil and the downward vertical). These angular relations are illustrated in Fig. 1.2.

Radiant flux, *Φ,* is the time rate of flow of radiant energy. It may be expressed in W ($\mathrm{J\,s^{-1}}$) or quanta $\mathrm{s^{-1}}$.

Radiant intensity, I, is a measure of the radiant flux per unit solid angle in a specified direction. The radiant intensity of a source in a given direction is the radiant flux emitted by a point source, or by an element of an extended source, in an infinitesimal cone containing the given direction, divided by that element of solid angle. We can also speak of radiant intensity at a point in space. This, the *field* radiant intensity, is the radiant flux at that point in a specified direction in an infinitesimal cone

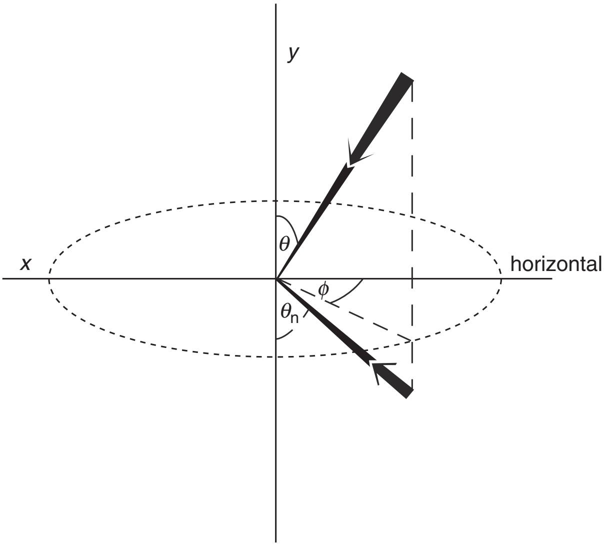

Fig. 1.2 The angles defining direction within a light field. The figure shows a downward and an upward pencil of light, both, for simplicity, in the same vertical plane. The downward pencil has zenith angle θ; the upward pencil has nadir angle θ_n, which is equivalent to a zenith angle of ($180° - \theta_n$). Assuming the *xy* plane is the vertical plane of the Sun, or other reference vertical plane, then ϕ is the azimuth angle for both light pencils.

containing the given direction, divided by that element of solid angle. *I* has the units W (or quanta s^{-1}) steradian^{-1}.

$$I = \mathrm{d}\Phi/\mathrm{d}\omega$$

If we consider the radiant flux not only per unit solid angle but also per unit area of a plane at right angles to the direction of flow, then we arrive at the even more useful concept of *radiance*, *L*. Radiance at a point in space is the radiant flux at that point in a given direction per unit solid angle per unit area at right angles to the direction of propagation. The meaning of this *field* radiance is illustrated in Figs. 1.3*a* and *b*. There is also *surface* radiance, which is the radiant flux emitted in a given direction per unit solid angle per unit projected area (apparent unit area, seen from the viewing direction) of a surface: this is illustrated in Fig. 1.3*c*. To indicate that it is a function of direction, i.e. of both zenith and azimuth angle, radiance is commonly written as $L(\theta, \phi)$. The angular structure of a light field is expressed in terms of the variation of radiance with θ and ϕ. Radiance has the units W (or quanta s^{-1}) m^{-2} steradian^{-1}.

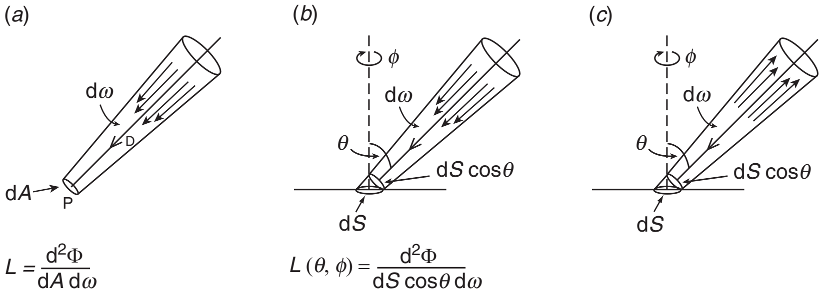

Fig. 1.3 Definition of radiance. (*a*) Field radiance at a point in space. The field radiance at P in the direction D is the radiant flux in the small solid angle surrounding D, passing through the infinitesimal element of area d*A* at right angles to D divided by the element of solid angle and the element of area. (*b*) Field radiance at a point in a surface. It is often necessary to consider radiance at a point on a surface, from a specified direction relative to that surface. d*S* is the area of a small element of surface. L(θ, ϕ) is the radiance incident on d*S* at zenith angle θ (relative to the normal to the surface) and azimuth angle ϕ: its value is determined by the radiant flux directed at d*S* within the small solid angle, dω, centred on the line defined by θ and ϕ. The flux passes perpendicularly across the area d*S* cos θ, which is the projected area of the element of surface, d*S*, seen from the direction θ, ϕ. Thus the radiance on a point in a surface, from a given direction, is the radiant flux in the specified direction per unit solid angle per unit projected area of the surface. (*c*) Surface radiance. In the case of a surface that emits radiation the intensity of the flux leaving the surface in a specified direction is expressed in terms of the surface radiance, which is defined in the same way as the field radiance at a point in a surface except that the radiation is considered to flow away from, rather than on to, the surface.

$$L(\theta, \phi) = \mathrm{d}^2\Phi/\mathrm{d}S\ \cos\theta\ \mathrm{d}\omega$$

Irradiance (at a point of a surface), *E*, is the radiant flux incident on an infinitesimal element of a surface, containing the point under consideration, divided by the area of that element. Less rigorously, it may be defined as the radiant flux per unit area of a surface.* It has the units $\mathrm{W\,m^{-2}}$ or quanta (or photons) $\mathrm{s^{-1}\,m^{-2}}$, or mol quanta (or photons) $\mathrm{s^{-1}\,m^{-2}}$, where 1.0 mol photons is 6.02×10^{23} (Avogadro's number) photons. One mole of photons is sometimes referred to as an *einstein,* but this term is now rarely used.

* Terms such as 'fluence rate' or 'photon fluence rate', sometimes to be found in the plant physiological literature, are superfluous and should not be used.

$$E = \mathrm{d}\Phi/\mathrm{d}S$$

Downward irradiance, E_d, and *upward irradiance,* E_u, are the values of the irradiance on the upper and the lower faces, respectively, of a horizontal plane. Thus, E_d is the irradiance due to the downwelling light stream and E_u is that due to the upwelling light stream.

The relation between irradiance and radiance can be understood with the help of Fig. 1.3*b*. The radiance in the direction defined by θ and ϕ is $L(\theta, \phi)$ W (or quanta s^{-1}) per unit projected area per steradian (sr). The projected area of the element of surface is dS cos θ and the corresponding element of solid angle is dω. Therefore the radiant flux on the element of surface within the solid angle dω is $L(\theta, \phi)\mathrm{d}S \cos\theta \,\mathrm{d}\omega$. The area of the element of surface is dS and so the irradiance at that point in the surface where the element is located, due to radiant flux within dω, is $L(\theta, \phi) \cos\theta \,\mathrm{d}\omega$. The total downward irradiance at that point in the surface is obtained by integrating with respect to solid angle over the whole upper hemisphere

$$E_d = \int_{2\pi} L(\theta, \phi) \cos\theta \,\mathrm{d}\omega \tag{1.5}$$

The total upward irradiance is related to radiance in a similar manner except that allowance must be made for the fact that cos θ is negative for values of θ between 90 and 180°

$$E_u = -\int_{-2\pi} L(\theta, \phi) \cos\theta \,\mathrm{d}\omega \tag{1.6}$$

Alternatively the cosine of the nadir angle, θ_n (see Fig. 1.2), rather than of the zenith angle, may be used

$$E_u = \int_{-2\pi} L(\theta_n, \phi) \cos\ \theta_n \,\mathrm{d}\omega \tag{1.7}$$

The -2π subscript is simply to indicate that the integration is carried out over the 2π sr solid angle in the *lower* hemisphere.

The *net downward irradiance*, $\vec{E}$, is the difference between the downward and the upward irradiance

$$\vec{E} = E_d - E_u \tag{1.8}$$

It is related to radiance by the eqn

$$\vec{E} = \int_{4\pi} L(\theta, \phi) \cos \theta \mathrm{d}\omega \tag{1.9}$$

which integrates the product of radiance and cos θ over all directions: the fact that cos θ is negative between 90 and 180 $^\circ$ ensures that the contribution of upward irradiance is negative in accordance with eqn 1.8. The net downward irradiance is a measure of the net rate of transfer of energy downwards at that point in the medium, and as we shall see later is a concept that can be used to arrive at some valuable conclusions.

The *scalar irradiance,* E_0, is the integral of the radiance distribution at a point over all directions about the point

$$E_o = \int_{4\pi} L(\theta, \phi) \mathrm{d}\omega \tag{1.10}$$

Scalar irradiance is thus a measure of the radiant intensity at a point, which treats radiation from all directions equally. In the case of irradiance, on the other hand, the contribution of the radiation flux at different angles varies in proportion to the cosine of the zenith angle of incidence of the radiation: a phenomenon based on purely geometrical relations (Fig. 1.3, eqn 1.5), and sometimes referred to as the Cosine Law. It is useful to divide the scalar irradiance into a downward and an upward component. The *downward scalar irradiance*, $E_{0\mathrm{d}}$, is the integral of the radiance distribution over the upper hemisphere

$$E_{0d} = \int_{2\pi} L(\theta, \phi) \mathrm{d}\omega \tag{1.11}$$

The *upward scalar irradiance is* defined in a similar manner for the lower hemisphere

$$E_{0u} = \int_{-2\pi} L(\theta, \phi) \mathrm{d}\omega \tag{1.12}$$

Scalar irradiance (total, upward, downward) has the same units as irradiance.

It is always the case in real-life radiation fields that irradiance and scalar irradiance vary markedly with wavelength across the photosynthetic range. This variation has a considerable bearing on the extent to which the radiation field can be used for photosynthesis. It is expressed in terms of the variation in irradiance or scalar irradiance per unit spectral distance (in units of wavelength or frequency, as appropriate) across the spectrum. Typical units would be W (or quanta s^{-1}) $\mathrm{m}^{-2}\,\mathrm{nm}^{-1}$.

If we know the radiance distribution over all angles at a particular point in a medium then we have a complete description of the angular structure of the light field. A complete radiance distribution, however, covering all zenith and azimuth angles at reasonably narrow intervals, represents a large amount of data: with 5° angular intervals, for example, the distribution will consist of 1369 separate radiance values. A simpler, but still very useful, way of specifying the angular structure of a light field is in the form of the three average cosines – for downwelling, upwelling and total light – and the irradiance reflectance.

The average cosine for downwelling light, $\overline{\mu}_d$, at a particular point in the radiation field, may be regarded as the average value, in an infinitesimally small volume element at that point in the field, of the cosine of the zenith angle of all the downwelling photons in the volume element. It can be calculated by summing (i.e. integrating) for all elements of solid angle (dω) comprising the upper hemisphere, the product of the radiance in that element of solid angle and the value of cos θ (i.e. $L(\theta, \phi)\cos\theta$), and then dividing by the total radiance originating in that hemisphere. By inspection of eqns 1.5 and 1.11 it can be seen that

$$\overline{\mu}_d = E_d/E_{0d} \tag{1.13}$$

i.e. the average cosine for downwelling light is equal to the downward irradiance divided by the downward scalar irradiance. The average cosine for upwelling light, $\overline{\mu}_u$, may be regarded as the average value of the cosine of the nadir angle of all the upwelling photons at a particular point in the field. By a similar chain of reasoning to the above, we conclude that $\overline{\mu}_u$ is equal to the upward irradiance divided by the upward scalar irradiance

$$\overline{\mu}_u = E_u/E_{0u} \tag{1.14}$$

In the case of the downwelling light stream it is often useful to deal in terms of the reciprocal of the average downward cosine, referred to by Preisendorfer (1961) as the *distribution function* for downwelling light, D_d, which can be shown[712] to be equal to the mean pathlength per vertical metre traversed, of the downward flux of photons per unit horizontal area per second. Thus $D_d = 1/\overline{\mu}_d$. There is, of course, an analogous distribution function for the upwelling light stream, defined by $D_u = 1/\overline{\mu}_u$.

The average cosine, $\overline{\mu}$, for the total light at a particular point in the field may be regarded as the average value, in an infinitesimally small volume element at that point in the field, of the cosine of the zenith angle of all the photons in the volume element. It may be evaluated by integrating the product of radiance and cos θ over all directions and dividing by the total radiance from all directions. By inspection of eqns 1.8, 1.9 and 1.10, it can be seen that the average cosine for the total light is equal to the *net* downward irradiance divided by the scalar irradiance

$$\overline{\mu} = \frac{\vec{E}}{E_0} = \frac{E_d - E_u}{E_0} \tag{1.15}$$

That $E_d - E_u$ should be involved (rather than, say, $E_d + E_u$) follows from the fact that the cosine of the zenith angle is negative for all the upwelling photons ($90° < \theta < 180°$). Thus a radiation field consisting of equal numbers of downwelling photons at $\theta = 45°$ and upwelling photons at $\theta = 135°$ would have $\overline{\mu} = 0$.

Average cosine is often written as $\overline{\mu}(z)$ to indicate that it is a function of the local radiation field at depth z. The total radiation field present in the water column also has an average cosine, $\overline{\mu}_c$, this being the average value of the cosine of the zenith angle of all the photons present in the water column at a given time.[716] In principle it could be evaluated by multiplying the value of $\overline{\mu}(z)$ in each depth interval by the proportion of the total water column radiant energy occurring in that depth interval, and then summing to obtain the average cosine for the whole water column, i.e. we would be making use of the relationship

$$\overline{\mu}_c = \int_0^\infty \overline{\mu}(z) \left[\frac{U(z)}{\int_0^\infty U(z) dz} \right] \mathrm{d}z \tag{1.16}$$

where $U(z)$ is the radiant energy density at depth z. The radiant energy density at depth z is equal to the scalar irradiance at that depth divided by the speed of light in water, c_{w}

$$U(z) = E_0(z)/c_{\mathrm{w}} \tag{1.17}$$

Making use of the fact that $\overline{\mu}(z)$ at any depth is equal to the net downward irradiance divided by the scalar irradiance (eqn 1.15), then substituting for $\overline{\mu}(z)$ and $U(z)$ in eqn 1.16 and cancelling out, we obtain

$$\overline{\mu}_c = \frac{\int_0^\infty [E_d(z) - E_u(z)] \mathrm{d}z}{\int_0^\infty E_0(z) \mathrm{d}z} \tag{1.18}$$

Taking eqns 1.16 to 1.18 to constitute an alternative definition of $\overline{\mu}_c$, then an appropriate alternative name for the average cosine of all the photons in the water column would be the *integral average cosine* of the underwater light field.

The remaining parameter that provides information about the angular structure of the light field is the *irradiance reflectance* (sometimes called the *irradiance ratio*), R. It is the ratio of the upward to the downward irradiance at a given point in the field

$$R = E_u/E_d \tag{1.19}$$

In any absorbing and scattering medium, such as sea or inland water, all these properties of the light field change in value with depth (for which we use the symbol z): the change might typically be a decrease, as in the case of irradiance, or an increase, as in the case of reflectance. It is sometimes useful to have a measure of the rate of change of any given property with depth. All the properties with which we have dealt that have the dimensions of radiant flux per unit area, diminish in value, as we shall see later, in an approximately exponential manner with depth. It is convenient with these properties to specify the rate of change of the logarithm of the value with depth since this will be approximately the same at all depths. In this way we may define the *vertical attenuation coefficient* for downward irradiance

$$K_d = -\frac{\mathrm{d}\ln E_d}{\mathrm{d}z} = -\frac{1}{E_d}\frac{\mathrm{d}E_d}{\mathrm{d}z} \tag{1.20}$$

upward irradiance

$$K_u = -\frac{\mathrm{d}\ln E_u}{\mathrm{d}z} = -\frac{1}{E_u}\frac{\mathrm{d}E_u}{\mathrm{d}z} \tag{1.21}$$

net downward irradiance

$$K_E = -\frac{\mathrm{d}\ln(E_d - E_u)}{\mathrm{d}z} = -\frac{1}{(E_d - E_u)}\frac{\mathrm{d}(E_d - E_u)}{\mathrm{d}z} \tag{1.22}$$

scalar irradiance

$$K_0 = -\frac{\mathrm{d}\ln E_0}{\mathrm{d}z} = -\frac{1}{E_o}\frac{\mathrm{d}E_0}{\mathrm{d}z} \tag{1.23}$$

radiance

$$K(\theta,\phi) = -\frac{\mathrm{d}\ln L(\theta,\phi)}{\mathrm{d}z} = -\frac{1}{L(\theta,\phi)}\frac{\mathrm{d}L(\theta,\phi)}{\mathrm{d}z} \tag{1.24}$$

In recognition of the fact that the values of these vertical attenuation coefficients are to some extent a function of depth they may sometimes be written in the form $K(z)$. For practical oceanographic and limnological purposes it is often desirable to have an estimate of the average value of a vertical attenuation coefficient in that upper layer (the *euphotic zone*) where light intensity is sufficient for significant photosynthesis to take place. A commonly used procedure is to calculate the linear regression coefficient of ln $E(z)$ with respect to depth over the depth interval of interest (§5.1). Choice of the most appropriate depth interval is inavoidably somewhat arbitrary. An alternative approach is to use the irradiance values themselves to weight the estimates of the irradiance attenuation coefficients.[717] This yields K values applicable to that part of the water column where most of the energy is attenuated. If we indicate the irradiance-weighted vertical attenuation coefficient by ${}^{w}K(\mathrm{av})$ then

$$ {}^{w}K(av) = \frac{\int_0^\infty K(z)E(z)\mathrm{d}z}{\int_0^\infty E(z)\mathrm{d}z} \tag{1.25} $$

where $E(z)$ can be $E_d(z)$, $E_u(z)$, $\vec{E}(z)$, or $E_0(z)$ and $K(z)$ can be $K_d(z)$, $K_u(z)$, $K_E(z)$ or $K_0(z)$, respectively. The meaning of eqn 1.25 is that when we calculate an average value of K by integrating over depth, at every depth the localized value of $K(z)$ is weighted by the appropriate value of the relevant type of irradiance at that depth. The integrated product of $K(z)$ and $E(z)$ over all depths is divided by the integrated irradiance over all depths.

1.4 The inherent optical properties

There are only two things that can happen to photons within water: they can be absorbed or they can be scattered. Thus if we are to understand what happens to solar radiation as it passes into any given water body, we need some measure of the extent to which that water absorbs and scatters light. The absorption and scattering properties of the aquatic medium for light of any given wavelength are specified in terms of the absorption coefficient, the scattering coefficient and the volume scattering function. These have been referred to by Preisendorfer (1961) as *inherent* optical properties (IOP), because their magnitudes depend only on the substances comprising the aquatic medium and not on the geometric structure of the light fields that may pervade it. They are defined with the help of an imaginary, infinitesimally thin, plane parallel layer of medium, illuminated at right angles by a parallel beam of monochromatic light (Fig. 1.4). Some of the incident light is absorbed by the thin layer. Some is

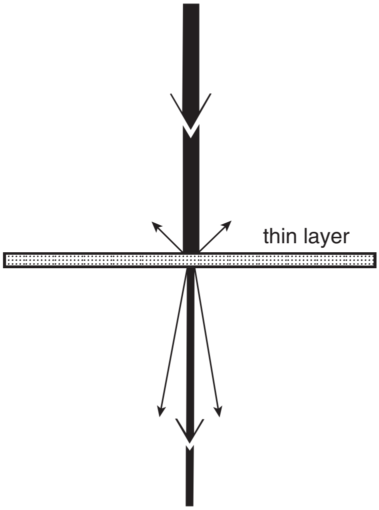

Fig. 1.4 Interaction of a beam of light with a thin layer of aquatic medium. Of the light that is not absorbed, most is transmitted without deviation from its original path: some light is scattered, mainly in a forward direction.

scattered – that is, caused to diverge from its original path. The fraction of the incident flux that is absorbed, divided by the thickness of the layer, is the *absorption coefficient*, a. The fraction of the incident flux that is scattered, divided by the thickness of the layer, is the *scattering coefficient*, b.

To express the definitions quantitatively we make use of the quantities *absorptance*, A, and *scatterance*, B. If Φ_0 is the radiant flux (energy or quanta per unit time) incident in the form of a parallel beam on some physical system, Φ_a is the radiant flux absorbed by the system, and Φ_b is the radiant flux scattered by the system. Then

$$A = \Phi_a/\Phi_0 \tag{1.26}$$

and

$$B = \Phi_b/\Phi_0 \tag{1.27}$$

i.e. absorptance and scatterance are the fractions of the radiant flux lost from the incident beam, by absorption and scattering, respectively. The sum of absorptance and scatterance is referred to as *attenuance*, C: it is the fraction of the radiant flux lost from the incident beam by absorption and scattering combined. In the case of the infinitesimally thin layer, thickness Δr, we represent the very small fractions of the incident flux that are lost by absorption and scattering as ΔA and ΔB, respectively. Then

$$a = \Delta A/\Delta r \tag{1.28}$$

and

$$b = \Delta B/\Delta r \tag{1.29}$$

An additional inherent optical property that we may now define is the *beam attenuation coefficient*, *c*. It is given by

$$c = a + b \tag{1.30}$$

and is the fraction of the incident flux that is absorbed and scattered, divided by the thickness of the layer. If the very small fraction of the incident flux that is lost by absorption and scattering combined is given the symbol ΔC (where $\Delta C = \Delta A + \Delta B$) then

$$c = \Delta C/\Delta r \tag{1.31}$$

The absorption, scattering and beam attenuation coefficients all have units of 1/length, and are normally expressed in m^{-1}.

In the real world we cannot carry out measurements on infinitesimally thin layers, and so if we are to determine the values of *a, b* and *c* we need expressions that relate these coefficients to the absorptance, scatterance and beam attenuance of layers of finite thickness. Consider a medium illuminated perpendicularly with a thin parallel beam of radiant flux, Φ_0. As the beam passes through, it loses intensity by absorption and scattering. Consider now an infinitesimally thin layer, thickness Δr, within the medium at a depth, *r*, where the radiant flux in the beam has diminished to Φ. The change in radiant flux in passing through Δr is $\Delta\Phi$. The attenuance of the thin layer is

$$\Delta C = -\Delta\Phi/\Phi$$

(the negative sign is necessary since $\Delta\Phi$ must be negative)

$$\Delta\Phi/\Phi = -c\Delta r$$

Integrating between 0 and *r* we obtain

$$\ln\frac{\Phi}{\Phi_0} = -cr \tag{1.32}$$

or

$$\Phi = \Phi_0\, e^{-cr} \tag{1.33}$$

indicating that the radiant flux diminishes exponentially with distance along the path of the beam. Equation 1.32 may be rewritten

$$c = \frac{1}{r} \ln \frac{\Phi_0}{\Phi} \tag{1.34}$$

or

$$c = -\frac{1}{r} \ln(1 - C) \tag{1.35}$$

The value of the beam attenuation coefficient, c, can therefore, using eqn 1.34 or 1.35, be obtained from measurements of the diminution in intensity of a parallel beam passing through a known pathlength of medium, r.

The theoretical basis for the measurement of the absorption and scattering coefficients is less simple. In a medium with absorption but negligible scattering, the relation

$$a = -\frac{1}{r} \ln(1 - A) \tag{1.36}$$

holds, and in a medium with scattering but negligible absorption, the relation

$$b = -\frac{1}{r} \ln(1 - B) \tag{1.37}$$

holds, but in any medium that both absorbs and scatters light to a significant extent, neither relation is true. This can readily be seen by considering the application of these equations to such a medium.

In the case of eqn 1.37 some of the measuring beam will be removed by absorption within the pathlength r before it has had the opportunity to be scattered, and so the amount of light scattered, B, will be lower than that required to satisfy the equation. Similarly, A will have a value lower than that required to satisfy eqn 1.36 since some of the light will be removed from the measuring beam by scattering before it has had the chance to be absorbed.

In order to actually measure a or b these problems must be circumvented. In the case of the absorption coefficient, it is possible to arrange that most of the light scattered from the measuring beam still passes through approximately the same pathlength of medium and is collected by the detection system. Thus the contribution of scattering to total attenuation is made very small and eqn 1.36 may be used. In the case of the scattering coefficient there is no instrumental way of avoiding the losses due to absorption and so the absorption must be determined separately and appropriate corrections made to the scattering data. We shall consider ways of measuring a and b in more detail later (§§ 3.2 and 4.2).

The way in which scattering affects the penetration of light into the medium depends not only on the value of the scattering coefficient but also on the angular distribution of the scattered flux resulting from the primary scattering process. This angular distribution has a characteristic shape for any given medium and is specified in terms of the *volume scattering function*, $\beta(\theta)$. This is defined as the radiant intensity in a given direction from a volume element, dV illuminated by a parallel beam of light, per unit of irradiance on the cross-section of the volume, and per unit volume (Fig. 1.5*a*). The definition is usually expressed mathematically in the form

$$\beta(\theta) = \mathrm{d}I(\theta)/E\ \mathrm{d}V \tag{1.38}$$

Since, from the definitions in §1.3

$$\mathrm{d}I(\theta) = \mathrm{d}\Phi(\theta)/\mathrm{d}\omega$$

and

$$E = \Phi_0/\mathrm{d}S$$

where d$\Phi(\theta)$ is the radiant flux in the element of solid angle dω, oriented at angle θ to the beam, and Φ_0 is the flux incident on the cross-sectional area, dS, and since

$$\mathrm{d}V = \mathrm{d}S.\mathrm{d}r$$

where dr is the thickness of the volume element, then we may write

$$\beta(\theta) = \frac{\mathrm{d}\Phi(\theta)}{\Phi_0}\frac{1}{d\ \omega \mathrm{d}r} \tag{1.39}$$

The volume scattering function has the units $\mathrm{m}^{-1}\,\mathrm{sr}^{-1}$.

Light scattering from a parallel light beam passing through a thin layer of medium is radially symmetrical around the direction of the beam. Thus, the light scattered at angle θ should be thought of as a cone with half-angle θ, rather than as a pencil of light (Fig. 1.5*b*).

From eqn 1.39 we see that $\beta(\theta)$ is the radiant flux per unit solid angle scattered in the direction θ, per unit pathlength in the medium, expressed as a proportion of the incident flux. The angular interval θ to $\theta + \Delta\theta$ corresponds to an element of solid angle equal to $2\pi \sin \theta\ \Delta\theta$ (Fig. 1.5*b*) and so the proportion of the incident radiant flux scattered (per unit pathlength) in this angular interval is $\beta(\theta)\ 2\pi \sin \theta\ \Delta\theta$. To obtain the proportion of the incident flux that is scattered in

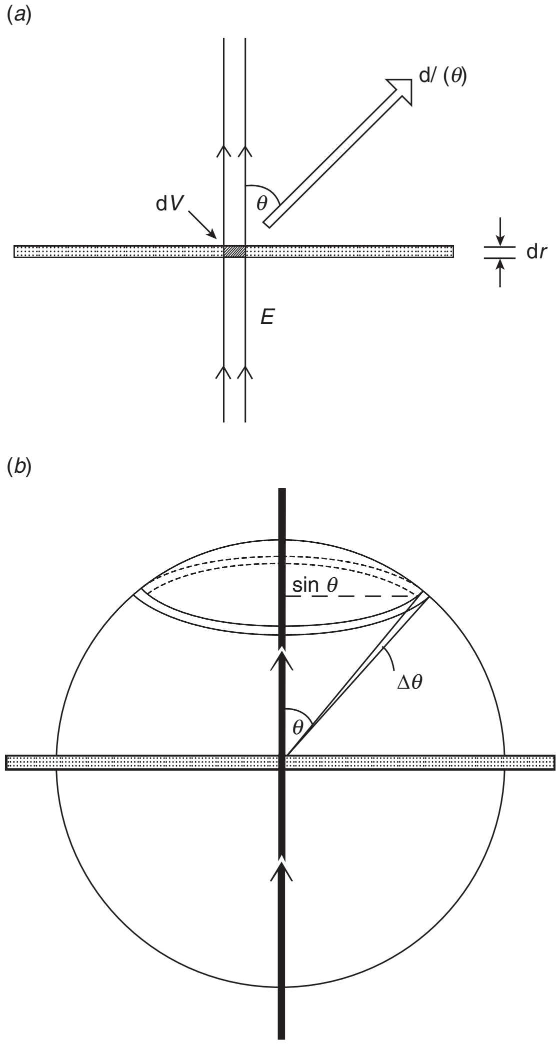

Fig. 1.5 The geometrical relations underlying the volume scattering function. (*a*) A parallel light beam of irradiance E and cross-sectional area dA passes through a thin layer of medium, thickness dr. The illuminated element of volume is dV. d$I(\theta)$ is the radiant intensity due to light scattered at angle θ. (*b*) The point at which the light beam passes through the thin layer of medium can be imagined as being at the centre of a sphere of unit radius. The light scattered between θ and $\theta + \Delta\theta$ illuminates a circular strip, radius $\sin\theta$ and width $\Delta\theta$, around the surface of the sphere. The area of the strip is $2\pi \sin\theta\, \Delta\theta$, which is equivalent to the solid angle (in steradians) corresponding to the angular interval, $\Delta\theta$.

all directions per unit pathlength – by definition, equal to the scattering coefficient – we must integrate over the angular range $\theta = 0°$ to $\theta = 180°$

$$b = 2\pi \int_0^{\pi} \beta(\theta) \sin\ \theta \mathrm{d}\theta = \int_{4\pi} \beta(\theta) \mathrm{d}\omega \quad (1.40)$$

Thus an alternative definition of the scattering coefficient is the integral of the volume scattering function over all directions.

It is frequently useful to distinguish between scattering in a forward direction and that in a backward direction. We therefore partition the total scattering coefficient, b, into a *forward scattering coefficient,* b_f, relating to light scattered from the beam in a forward direction, and a *backward scattering coefficient* (or simply, *backscattering coefficient*) b_b, relating to light scattered from the beam in a backward direction

$$b = b_b + b_f \quad (1.41)$$

We may also write

$$b_f = 2\pi \int_0^{\pi/2} \beta(\theta) \sin\theta \mathrm{d}\theta \quad (1.42)$$

$$b_b = 2\pi \int_{\pi/2}^{\pi} \beta(\theta) \sin\theta \mathrm{d}\theta \quad (1.43)$$

The variation of $\beta(\theta)$ with θ tells us the absolute amount of scattering at different angles, per unit pathlength in a given medium. If we wish to compare the *shape* of the angular distribution of scattering in different media separately from the absolute amount of scattering that occurs, then it is convenient to use the *normalized volume scattering function*, $\tilde{\beta}(\theta)$, sometimes called the scattering phase function, which is that function (units sr^{-1}) obtained by dividing the volume scattering function by the total scattering coefficient

$$\tilde{\beta}(\theta) = \beta(\theta)/b \quad (1.44)$$

The integral of $\tilde{\beta}(\theta)$over all solid angles is equal to 1. The integral of $\tilde{\beta}(\theta)$up to any given value of θ is the proportion of the total scattering that occurs in the angular interval between $0°$ and that value of θ. We can also define normalized forward scattering and backward scattering coefficients, $\tilde{b}_f$ and $\tilde{b}_b$, as the proportions of the total scattering in forwards and backwards directions, respectively

$$\tilde{b}_f = b_f/b \tag{1.45}$$

$$\tilde{b}_b = b_b/b \tag{1.46}$$

Just as it is useful sometimes to express the angular structure of a light field in terms of a single parameter – its average cosine ($\overline{\mu}$) – so it can also be useful in the case of the scattering phase function to have a single parameter that provides some indication of its shape. Such a parameter is the *average cosine of scattering*, $\overline{\mu}_s$, which can be thought of as the average cosine of the singly scattered light field. It is also sometimes referred to as the *asymmetry factor*, and given the symbol, g. Its value, for any given volume scattering function, may be calculated[712] from

$$\overline{\mu}_s = \frac{\int_{4\pi} \beta(\theta)\cos\theta \mathrm{d}\omega}{\int_{4\pi} \beta(\theta)\mathrm{d}\omega} \tag{1.47}$$

or (using eqn 1.44 and the fact that the integral of $\tilde{\beta}(\theta)$ over 4π is 1) from

$$\overline{\mu}_s = \int_{4\pi} \tilde{\beta}(\theta)\cos\theta \mathrm{d}\omega \tag{1.48}$$

1.5 Apparent and quasi-inherent optical properties

The vertical attenuation coefficients for radiance, irradiance and scalar irradiance are, strictly speaking, properties of the radiation field since, by definition, each of them is the logarithmic derivative with respect to depth of the radiometric quantity in question. Nevertheless experience has shown that their values are largely determined by the inherent optical properties of the aquatic medium and are not very much altered by changes in the incident radiation field such as a change in solar elevation.[59] For example, if a particular water body is found to have a high value of K_d then we expect it to have approximately the same high K_d tomorrow, or next week, or at any time of the day, so long as *the composition of the water remains about the same*.

Vertical attenuation coefficients, such as K_d, are thus commonly used, and thought of, by oceanographers and limnologists as though they are optical properties belonging to the water, properties that are a direct measure of the ability of that water to bring about a diminution in the appropriate radiometric quantity with depth. Furthermore they have the same units (m^{-1}) as the inherent optical properties a, b

and c. In recognition of these useful aspects of the various K functions, Preisendorfer (1961) suggested that they be classified as *apparent optical properties* (AOP) and we shall so treat them in this book. The reflectance, R, is also often treated as an apparent optical property of water bodies.

The two fundamental inherent optical properties – the coefficients for absorption and scattering – are, as we saw earlier, defined in terms of the behaviour of a parallel beam of light incident upon a thin layer of medium. Analogous coefficients can be defined for incident light streams having any specified angular distribution. In particular, such coefficients can be defined for incident light streams corresponding to the upwelling and downwelling streams that exist at particular depths in real water bodies. We shall refer to these as the *diffuse* absorption and scattering coefficients for the upwelling or downwelling light streams at a given depth. Although related to the normal coefficients, the values of the diffuse coefficients are a function of the local radiance distribution, and therefore of depth.

The *diffuse absorption coefficient* for the downwelling light stream at depth z, $a_d(z)$, is the proportion of the incident radiant flux that would be absorbed from the downwelling stream by an infinitesimally thin horizontal plane parallel layer at that depth, divided by the thickness of the layer. The diffuse absorption coefficient for the upwelling stream, $a_u(z)$, is defined in a similar way. Absorption of a diffuse light stream within the thin layer will be greater than absorption of a normally incident parallel beam because the pathlengths of the photons will be in proportion to $1/\overline{\mu}_d$ and $1/\overline{\mu}_u$, respectively. The diffuse absorption coefficients are therefore related to the normal absorption coefficients by

$$a_d(z) = \frac{a}{\overline{\mu}_d(z)} \tag{1.49}$$

$$a_u(z) = \frac{a}{\overline{\mu}_u(z)} \tag{1.50}$$

where $\overline{\mu}_d(z)$ and $\overline{\mu}_u(z)$ are the values of $\overline{\mu}_d$ and $\overline{\mu}_u$ that exist at depth z.

So far as scattering of the upwelling and downwelling light streams is concerned, it is mainly the backward scattering component that is of importance. The diffuse backscattering coefficient for the downwelling stream at depth z, $b_{bd}(z)$, is the proportion of the incident radiant flux from the downwelling stream that would be scattered backwards (i.e. upwards) by an infinitesimally thin, horizontal plane parallel layer at that depth, divided by the thickness of the layer: $b_{bu}(z)$, the corresponding coefficient for the upwelling stream is defined in the same way in terms of the light scattered downwards again from that stream. Diffuse total ($b_d(z)$, $b_u(z)$) and forward ($b_{fd}(z)$, $b_{fu}(z)$) scattering coefficients for the downwelling and upwelling streams can be defined in a similar manner. The following relations hold

$$b_d(z) = b/\overline{\mu}_d(z), \qquad b_u(z) = b/\overline{\mu}_u(z)$$
$$b_d(z) = b_{fd}(z) + b_{bd}(z), \quad b_u(z) = b_{fu}(z) + b_{bu}(z)$$

The relation between a diffuse backscattering coefficient and the normal backscattering coefficient, b_b, is not simple but may be calculated from the volume scattering function and the radiance distribution existing at depth z. The calculation procedure is discussed later (§ 4.2).

Preisendorfer (1961) has classified the diffuse absorption and scattering coefficients as *hybrid optical properties* on the grounds that they are derived both from the inherent optical properties and certain properties of the radiation field. I prefer the term *quasi-inherent optical properties,* on the grounds that it more clearly indicates the close relation between these properties and the inherent optical properties. Both sets of properties have precisely the same kind of definition: they differ only in the characteristics of the light flux that is imagined to be incident upon the thin layer of medium.

The important quasi-inherent optical property, $b_{bd}(z)$, can be linked with the two apparent optical properties, K_d and R, with the help of one more optical property, κ(z), which is the average vertical attenuation coefficient in upward travel from their first point of upward scattering, of all the upwelling photons received at depth z.[710] κ(z) must not be confused with, and is in fact much greater than, $K_u(z)$, the vertical attenuation coefficient (with respect to depth increasing downward) of the upwelling light stream. Using κ(z) we link the apparent and the quasi-inherent optical properties in the relation

$$R(z) \approx \frac{b_{bd}(z)}{K_d(z) + \kappa(z)} \tag{1.51}$$

At depths where the asymptotic radiance distribution is established (see § 6.6) this relationship holds exactly. Monte Carlo modelling of the underwater light field for a range of optical water types[710] has shown that κ is approximately linearly related to K_d, the relationship at z_m (a depth at which irradiance is 10% of the subsurface value) being

$$\kappa(z_m) \approx 2.5\, K_d(z_m) \tag{1.52}$$

1.6 Optical depth

As we have already noted, but will discuss more fully later, the downward irradiance diminishes in an approximately exponential manner with depth. This may be expressed by the equation

$$E_d(z) = E_d(0)e^{-K_d Z} \tag{1.53}$$

where $E_d(z)$ and $E_d(0)$ are the values of downward irradiance at z m depth, and just below the surface, respectively, and K_d is the average value of the vertical attenuation coefficient over the depth interval 0 to z m. We shall now define the *optical depth*, ζ, by the eqn

$$\zeta = K_d z \tag{1.54}$$

It can be seen that a specified optical depth will correspond to different physical depths but to the same overall diminution of irradiance, in waters of differing optical properties. Thus in a coloured turbid water with a high K_d, a given optical depth will correspond to a much smaller actual depth than in a clear colourless water with a low K_d. Optical depth, ζ, as defined here is distinct from *attenuation length*, τ (sometimes also called optical depth or optical distance), which is the geometrical length of a path multiplied by the beam attenuation coefficient (c) associated with the path.

Optical depths of particular interest in the context of primary production are those corresponding to attenuation of downward irradiance to 10% and 1% of the subsurface values: these are $\zeta = 2.3$ and $\zeta = 4.6$, respectively. These optical depths correspond to the mid-point and the lower limit of the euphotic zone, within which significant photosynthesis occurs.

1.7 Radiative transfer theory

Having defined the properties of the light field and the optical properties of the medium we are now in a position to ask whether it is possible to arrive, on purely theoretical grounds, at any relations between them. Although, given a certain incident light field, the characteristics of the underwater light field are uniquely determined by the properties of the medium, it is nevertheless true that explicit, all-embracing analytical relations, expressing the characteristics of the field in terms of the inherent optical properties of the medium, have not yet been derived. Given the complexity of the shape of the volume scattering function in natural waters (see Chapter 4), it may be that this will never be achieved.

It is, however, possible to arrive at a useful expression relating the absorption coefficient to the average cosine and the vertical attenuation coefficient for net downward irradiance. In addition, relations have been derived between certain properties of the field and the diffuse optical properties. These various relations are all arrived at by making use of the equation of transfer for radiance. This describes the manner in which radiance varies with distance along any specified path at a specified point in the medium.

Assuming a horizontally stratified water body (i.e. with properties everywhere constant at a given depth), with a constant input of monochromatic unpolarized radiation at the surface, and ignoring fluorescent emission within the water, the equation may be written

$$\frac{\mathrm{d}L(z,\theta,\phi)}{\mathrm{d}r} = -c(z)L(z,\theta,\phi) + L^*(z,\theta,\phi) \tag{1.55}$$

The term on the left is the rate of change of radiance with distance, r, along the path specified by zenith and azimuthal angles θ and ϕ, at depth z. The net rate of change is the resultant of two opposing processes: loss by attenuation along the direction of travel ($c(z)$ being the value of the beam attenuation coefficient at depth z), and gain by scattering (along the path dr) of light initially travelling in other directions (θ', ϕ') into the direction θ, ϕ (Fig. 1.6). This latter term is determined by the volume scattering function of the medium at depth z (which we write $\beta(z, \theta, \phi; \theta', \phi')$ to indicate that the scattering angle is the angle between the two directions θ, ϕ and θ', ϕ') and by the distribution of radiance, $L(z, \theta', \phi')$. Each element of irradiance, $L(z, \theta', \phi')\mathrm{d}\omega(\theta', \phi')$ (where $\mathrm{d}\omega(\theta', \phi')$ is an element of solid angle forming an infinitesimal cone containing the direction θ', ϕ'), incident on the volume element along dr gives rise to some scattered radiance in the direction θ, ϕ. The total radiance derived in this way is given by

$$L^*(z,\theta,\phi) = \int_{2\pi} \beta(z,\theta,\phi;\theta',\phi')\, L(z,\theta',\phi')\, \mathrm{d}\omega(\theta',\phi') \tag{1.56}$$

If we are interested in the variation of radiance in the direction θ, ϕ as a function of depth, then since dr = dz/cos θ, we may rewrite eqn 1.55 as

$$\cos\theta \frac{\mathrm{d}L(z,\theta,\phi)}{\mathrm{d}z} = -c(z)L(z,\theta,\phi) + L^*(z,\theta,\phi) \tag{1.57}$$

By integrating each term of this equation over all angles

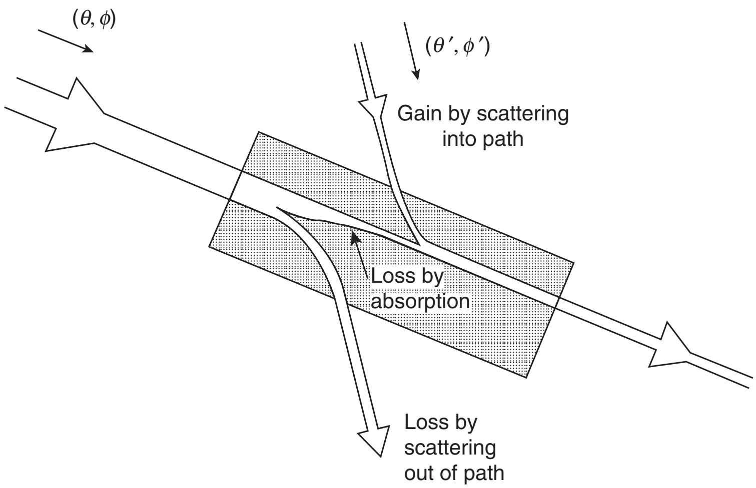

Fig. 1.6 The processes underlying the equation of transfer of radiance. A light beam passing through a distance, dr, of medium, in the direction θ, ϕ, loses some photons by scattering out of the path and some by absorption by the medium along the path, but also acquires new photons by scattering of light initially travelling in other directions (θ', ϕ') into the direction θ, ϕ.

$$\int_{4\pi} \cos\theta \frac{\mathrm{d}L(z,\theta,\phi)}{\mathrm{d}z}\,\mathrm{d}\omega = -\int_{4\pi} c(z)L(z,\theta,\phi)\mathrm{d}\omega + \int_{4\pi} L^*(z,\theta,\phi)\mathrm{d}\omega$$

we arrive at the relation

$$\frac{\mathrm{d}\vec{E}}{\mathrm{d}z} = -cE_0 + bE_0 = -aE_0 \tag{1.58}$$

originally derived by Gershun (1936).

It follows that

$$a = K_E \frac{\vec{E}}{E_0} \tag{1.59}$$

and

$$a = K_E \bar{\mu} \tag{1.60}$$

Thus we have arrived at a relation between an inherent optical property and two of the properties of the field. Equation 1.60, as we shall see later (§ 3.2), can be used as the basis for determining the absorption coefficient of a natural water from *in situ* irradiance and scalar irradiance measurements.

Exploration of the properties of irradiance-weighted vertical attenuation coefficients (defined in §1.3, above) has shown[717] that the following relationships, analogous to the Gershun equation, also exist

$$^{w}K_E(av) = \frac{a}{\overline{\mu}_c} \tag{1.61}$$

and

$$^{w}K_0(av) = \frac{a}{\overline{\mu}(0)} \tag{1.62}$$

where $^{w}K_E(av)$ and $^{w}K_0(av)$are the irradiance-weighted vertical attenuation coefficients for net downward and scalar irradiances, respectively, $\overline{\mu}_c$ is the integral average cosine for the whole water column, and $\overline{\mu}(0)$ is the average cosine for the light field just below the water surface.

Preisendorfer (1961) has used the equation of transfer to arrive at a set of relations between certain properties of the field and the *diffuse* absorption and scattering coefficients. One of these, an expression for the vertical attenuation coefficient for downward irradiance,

$$K_d(z) = a_d(z) + b_{bd}(z) - b_{bu}(z)R(z) \tag{1.63}$$

we will later (§ 6.7) find of assistance in understanding the relative importance of the different processes underlying the diminution of irradiance with depth.